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Practice 5-4

Factoring Quadratic Expression

Factor each expression completely.

1. $x^2 + 4x + 4$
2. $x^2 - 7x + 10$
3. $x^2 + 7x - 8$
4. $x^2 - 6x$
5. $2x^2 - 9x + 4$
6. $x^2 + 2x - 35$
7. $x^2 + 6x + 5$
8. $x^2 - 9$
9. $x^2 - 13x - 48$
10. $x^2 - 4$
11. $4x^2 + x$
12. $x^2 - 29x + 100$
13. $x^2 - x - 6$
14. $9x^2 - 1$
15. $3x^2 - 2x$
16. $x^2 - 64$
17. $x^2 - 25$
18. $x^2 - 81$
19. $x^2 - 36$
20. $x^2 - 100$
21. $x^2 - 1$
22. $4x^2 - 1$
23. $4x^2 - 36$
24. $9x^2 - 4$
25. $x^2 - 7x - 8$
26. $x^2 + 13x + 36$
27. $x^2 - 5x + 6$
28. $x^2 + 5x + 4$
29. $x^2 - 21x - 22$
30. $x^2 + 13x + 40$
31. $2x^2 - 5x - 3$
32. $x^2 + 10x - 11$
33. $x^2 - 14x + 24$
34. $5x^2 + 4x - 12$
35. $2x^2 - 5x - 7$
36. $2x^2 + 13x + 15$
37. $3x^2 - 7x - 6$
38. $3x^2 + 16x + 21$
39. $x^2 + 5x - 24$
40. $x^2 + 34x - 72$
41. $x^2 - 11x$
42. $3x^2 + 21x$
43. $x^2 + 8x + 12$
44. $x^2 - 10x + 24$
45. $x^2 + 7x - 30$
46. $x^2 - 2x - 168$
47. $x^2 - x - 72$
48. $4x^2 - 25$
49. $x^2 - 121$
50. $x^2 + 17x + 16$
51. $10x^2 - 17x + 3$
52. $4x^2 + 12x + 9$
53. $4x^2 - 4x - 15$
54. $9x^2 - 4$
55. $x^2 + 6x - 40$
56. $2x^2 - 8$
57. $x^2 + 18x + 77$
58. $2x^2 - 98$
59. $x^2 + 21x + 98$
60. $x^2 + 20x + 84$
61. $9x^2 + 30x + 16$
62. $8x^2 - 6x - 27$
63. $x^2 - 3x - 54$
64. $x^2 - 169$
65. $25x^2 - 9$
66. $7x^2 + 49$
67. $2x^2 - 10x - 28$
68. $x^2 + 8x + 12$
69. $x^2 - 2x - 35$
70. $x^2 + 2x - 63$
71. $20x^2 - 11x - 3$
72. $12x^2 + 4x - 5$
73. $4x^2 - 5x - 6$
74. $8x^2 + 22x - 21$
75. $3x^2 - 3x - 168$

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Practice 5-5

Quadratic Equations

Solve each equation by factoring, by taking square roots, or by graphing. When necessary, round your answer to the nearest hundredth.

1. $x^2 - 18x - 40 = 0$
2. $16x^2 = 56x$
3. $5x^2 = 15x$
4. $x^2 - 6x - 7 = 0$
5. $x^2 - 49 = 0$
6. $x^2 + 2x + 1 = 0$
7. $x^2 - 1 = 0$
8. $x^2 - 3x - 4 = 0$
9. $x^2 + 9x^2 + 20 = 0$
10. $6x^2 + 9 = -55x$
11. $(x + 5)^2 = 36$
12. $2x^2 - 3x = 0$
13. $2x^2 + x - 10 = 0$
14. $-4x^2 + 3x = -1$
15. $5x^2 - 6x + 1 = 0$
16. $3x^2 + 1 = -4x$
17. $-2x^2 + 2 = -3x$
18. $6x^2 + 1 = 5x$
19. $-2x^2 - x + 1 = 0$
20. $3x^2 + 5x = 2$
21. $x^2 - 6x = -8$
22. $x^2 + 6 = -7x$
23. $6x^2 + 18x = 0$
24. $2x^2 + 5 = 11x$
25. $3x^2 - 7x + 2 = 0$
26. $2x^2 - 3x = -1$
27. $2x^2 - x = 6$
28. $x^2 - 144 = 0$
29. $4x^2 + 2 = 6x$
30. $5x^2 + 2 = -7x$
31. $7x^2 + 6x - 1 = 0$
32. $2x^2 - 6x = -4$
33. $11x^2 - 12x + 1 = 0$
34. $7x^2 + 1 = -8x$
35. $x^2 + 9 = -10x$
36. $(x - 2)^2 = 18$
37. $x^2 - 8x + 7 = 0$
38. $x^2 - 16 = 0$
39. $x^2 + 6x = -8$
40. $x^2 + 3 = 4x$
41. $2x^2 + 6 = -7x$
42. $6x^2 + 2 = 7x$
43. $(x + 7)^2 = \frac{49}{16}$
44. $9x^2 - 8x = 1$
45. $10x^2 + 7x + 1 = 0$
46. $4x^2 + 2 = -9x$
47. $3x^2 + 4 = 8x$
48. $4x^2 + 5 + 9x = 0$
49. $9x^2 + 10x = -1$
50. $2x^2 + 9x + 4 = 0$
51. $2x^2 + 6x = -4$
52. $11x^2 - 1 = -10x$
53. $4x^2 = 1$
54. $6x^2 = 12x$
55. $25x^2 - 9 = 0$
56. $2x^2 + 11x = 6$
57. $8x^2 - 6x + 1 = 0$
58. $x^2 + 11 = -12x$
59. $6x^2 + 2 = 13x$
60. $x^2 = 121$
61. $4x^2 - 11x = 3$
62. $8x^2 + 6x + 1 = 0$
63. $x^2 + 9x + 8 = 0$
64. $x^2 + 8x = -12$
65. $x^2 + 6x = 40$
66. $2x^2 = 8$
67. $x^2 = x + 6$
68. $x^2 + 2x - 6 = 0$
69. $x^2 - 12 = 0$
70. $3x^2 + 4x = 6$
71. $7x^2 - 105 = 0$
72. $16x^2 = 81$
73. $x^2 + 5x + 4 = 0$
74. $x^2 + 36 = -13x$
75. $x^2 + 6 = 5x$

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Practice 5-7

Completing the Square

1. $x^2 + 6x +$ ■
2. $x^2 - 7x +$ ■
3. $x^2 + 12x +$ ■
4. $x^2 + 3x +$ ■
5. $x^2 - 8x +$ ■
6. $x^2 + 16x +$ ■
7. $x^2 + 21x +$ ■
8. $x^2 - 2x +$ ■

Rewrite each equation in vertex form. Then find the vertex.

9. $y = x^2 + 4x - 6$
10. $y = x^2 - 6x + 6$
11. $y = 4x^2 + 8x - 4$
12. $y = 4x^2 + 4x + 1$
13. $y = 2x^2 + 4x - 5$
14. $y = -3x^2 - 4x - 1$
15. $y = -3x^2 + 3x - 1$
16. $y = x^2 + 2x + 1$
17. $y = -5x^2 + 10x + 1$
18. $y = -2x^2 + 4x + 3$
19. $y = x^2 + 5x + \frac{5}{4}$
20. $y = -2x^2 + 10x - 11$
21. $y = 6x^2 - 12x + 1$
22. $y = -2x^2 + 8x - 9$
23. $y = 3x^2 + 9x + 6$

Solve each quadratic equation by completing the square.

24. $x^2 + 12x + 4 = 0$
25. $x^2 - x - 5 = 0$
26. $3x^2 = -12x - 3$
27. $x^2 - x - 1 = 0$
28. $4x^2 - 8x + 1 = 0$
29. $5x^2 = 8x - 6$
30. $2x^2 - 4x - 3 = 0$
31. $x^2 + 11x = 0$
32. $x^2 = 5x + 14$
33. $2x^2 + x - 1 = 0$
34. $2x^2 + 6x - 7 = 0$
35. $2x^2 = -8x + 45$
36. $x^2 = -3x - 3$
37. $4x^2 = -2x + 1$
38. $3x^2 = -6x + 9$
39. $x^2 = 7x + 12$
40. $x^2 = 3x + 7$
41. $3x^2 = 6x - 9$
42. $x^2 = -3x + 2$
43. $x^2 = -7x - 1$
44. $4x^2 = -3x + 2$
45. $2x^2 = 4x - 5$
46. $2x^2 = 5x + 5$
47. $2x^2 = 6x + 5$
48. $x^2 = 3x$
49. $x^2 = 8x$
50. $4x^2 = -2x - 3$
51. $2x^2 = -2x + 5$
52. $2x^2 = -5x - 5$
53. $3x^2 = -5x + 1$
54. $2x^2 = 2x + 4$
55. $3x^2 = 7x + 8$
56. $2x^2 = -6x + 4$
57. $x^2 = -7x - 9$
58. $2x^2 = 5x$
59. $3x^2 = -42x$
60. $2x^2 = -4x + 5$
61. $4x^2 = -x + 5$
62. $3x^2 = -3x + 1$
63. $x^2 = 3x + 4$
64. $2x^2 = 2x + 8$
65. $3x^2 = x + 4$

Solve each equation.

66. $x^2 + 2x + 1 = 9$
67. $3x^2 - 18x + 27 = 125$
68. $x^2 - 4x + 4 = 5$
69. $x^2 + 3x + \frac{9}{4} = \frac{13}{4}$
70. $x^2 + 3x + \frac{9}{4} = -\frac{15}{4}$
71. $x^2 + 3x + \frac{9}{4} = \frac{41}{4}$
72. $x^2 + 7x + \frac{49}{4} = \frac{53}{4}$
73. $x^2 + 3x + \frac{9}{4} = \frac{29}{4}$
74. $x^2 - 6x + 9 = 7$

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practice 5-8

The Quadratic Formula

Evaluate the discriminant of each equation. Tell how many solutions each equation has and whether the solutions are real or imaginary.

1. $y = x^2 + 10x - 25$
2. $y = x^2 + 10x + 10$
3. $y = 9x^2 - 24x$
4. $y = 4x^2 - 4x + 1$
5. $y = 4x^2 - 5x + 1$
6. $y = 4x^2 - 3x + 1$
7. $y = x^2 + 3x + 4$
8. $y = x^2 + 7x - 3$
9. $y = -2x^2 + 3x - 5$
10. $y = x^2 - 5x + 4$
11. $y = x^2 + 12x + 36$
12. $y = x^2 + 2x + 3$
13. $y = 2x^2 - 13x - 7$
14. $y = -5x^2 + 6x - 4$
15. $y = -4x^2 - 4x - 1$

Solve each equation using the Quadratic Formula.

16. $x^2 + 6x + 9 = 0$
17. $x^2 - 15x + 56 = 0$
18. $3x^2 - 5x + 2 = 0$
19. $2x^2 + 3x + 5 = 0$
20. $10x^2 - 23x + 12 = 0$
21. $4x^2 + x - 5 = 0$
22. $x^2 + 8x + 15 = 0$
23. $3x^2 + 2x + 1 = 0$
24. $4x^2 + x + 5 = 0$
25. $x^2 - 4x - 12 = 0$
26. $x^2 = 3x + 2$
27. $2x^2 - 5x + 2 = 0$
28. $x^2 + 6x - 4 = 0$
29. $x^2 = 2x - 5$
30. $3x^2 + 7 = -6x$
31. $2x^2 + 6x + 3 = 0$
32. $x^2 = -18x - 80$
33. $x^2 + 9x - 13 = 0$
34. $x^2 - 8x + 25 = 0$
35. $4x^2 + 13x = 12$
36. $3x^2 - 5x = -12$
37. $3x^2 + 4x + 5 = 0$
38. $2x^2 = 3x - 7$
39. $5x^2 + 2x + 1 = 0$
40. $5x^2 + x + 3 = 0$
41. $5x^2 + x = 3$
42. $5x^2 - 2x + 7 = 0$
43. $x^2 - 2x + 3 = 0$
44. $-2x^2 + 3x = 24$
45. $4x^2 = 5x - 6$
46. $x^2 + 6x + 5 = 0$
47. $x^2 - 6x = -8$
48. $x^2 - 6x = -6$

Solve.

49. A model of the daily profits p of a gas station based on the price per gallon g is $p = -15{,}000g^2 + 34{,}500g - 16{,}800$. Use the discriminant to find whether the station can profit \$4000 per day. Explain.

Solve each equation using the Quadratic Formula. Find the exact solutions. Then approximate any radical solutions. Round to the nearest hundredth.

50. $x^2 - 2x - 3 = 0$
51. $x^2 + 5x + 4 = 0$
52. $x^2 - 2x - 8 = 0$
53. $7x^2 - 12x + 3 = 0$
54. $5x^2 + 5x - 1 = 0$
55. $4x^2 + 5x + 1 = 0$
56. $6x^2 + 5x - 4 = 0$
57. $x^2 + x = 6$
58. $x^2 - 13x = 48$
59. $2x^2 + 5x = 0$
60. $x^2 + 3x - 3 = 0$
61. $x^2 - 4x + 1 = 0$
62. $9x^2 - 6x - 7 = 0$
63. $x^2 - 35 = 2x$
64. $x^2 + 7x + 10 = 0$

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A complex number plane can be used to _____

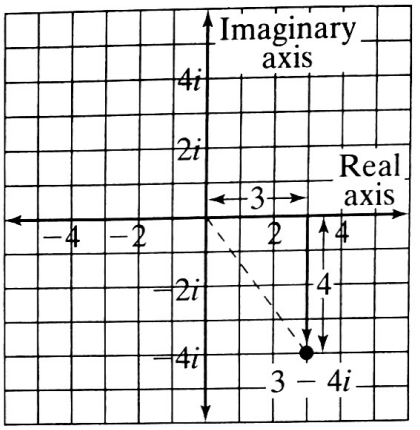
_____

_____

The absolute value of a complex number is _____

_____

_____



Examples

❶ Simplifying Numbers Using *i* Simplify $\sqrt{-54}$ by using the imaginary number i.

$\sqrt{-54} = \sqrt{\square} \cdot 54$

$= \sqrt{\square} \cdot \sqrt{54}$

$= \square \cdot \sqrt{54}$

$= \square \cdot \square\sqrt{\square}$

$= \square$

❷ Simplifying Imaginary Numbers Write the complex number $\sqrt{-121} - 7$ in the form $a + bi$.

$\sqrt{-121} - 7 = \square - 7$ **Simplify the radical expression.**

$= \square + \square$ **Write in the form *a* + *bi*.**

Quick Check

1. Simplify each number by using the imaginary number i.

a. $\sqrt{-2}$

b. $\sqrt{-12}$

c. $\sqrt{-36}$

2. Write the complex number $\sqrt{-18} + 7$ in the form $a + bi$.

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Lesson 5-8

The Quadratic Formula

Lesson Objectives	NAEP 2005 Strand: Algebra
1 Solving quadratic equations by using the Quadratic Formula	Topic: Equations and Inequalities
2 Determining types of solutions by using the discriminant	Local Standards: _____

Key Concepts

Quadratic Formula

A quadratic equation written in standard form $ax^2 + bx + c = 0$ can be solved with the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{\square - 4ac}}{\square}$$

Discriminant of a Quadratic Equation

The discriminant of a quadratic equation in the form $ax^2 + bx + c = 0$ is the value of the expression $b^2 - 4ac$.

$$x = \frac{-b \pm \sqrt{\square}}{2a} \leftarrow \textbf{discriminant}$$

Methods for Solving Quadratic Equations

Discriminant	Methods
positive square number	• factoring, graphing, Quadratic Formula, or completing the square
positive nonsquare number	• for approximate solutions: graphing, Quadratic Formula, or completing the square • for exact solutions: Quadratic Formula or completing the square
zero	• factoring, graphing, Quadratic Formula, or completing the square
negative	• Quadratic Formula or completing the square

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Examples

1 Using the Quadratic Formula The longer leg of a right triangle is 1 unit longer than the shorter leg. The hypotenuse is 3 units long. What is the length of the shorter leg?

$x^2 + (\square + \square)^2 = \square$ — Use the Pythagorean Theorem.

$x^2 + \square + \square + \square = \square$

$\square + 2x - \square = 0$ — Write in standard form.

$a = \square, b = 2, c = \square$ — Find the values of *a*, *b*, and *c*.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the Quadratic Formula.

$= \dfrac{\square \pm \sqrt{(\square)^2 - 4(2)(\square)}}{2(\square)}$ — Substitute for *a*, *b*, and *c*.

$= \dfrac{-\square \pm \sqrt{\square}}{\square}$ — Simplify.

The length of the shorter leg is $\dfrac{\sqrt{17} - 1}{2}$ units.

Check Is the answer reasonable? Since $\dfrac{-1 - \sqrt{17}}{2}$ is a [ ] number, and a length cannot be negative, that answer is [ ].

Since $\dfrac{\sqrt{17} - 1}{2} \approx$ [ ], that answer is [ ].

2 Using the Discriminant Determine the type and number of solutions of $x^2 + 5x + 10 = 0$.

$a = \square, b = \square, c = \square$ — Find the values of *a*, *b*, and *c*.

$b^2 - 4ac = (\square)^2 - 4(\square)(\square)$ — Evaluate the discriminant.

$= 25 - \square = \square$ — Simplify.

Since the disciminant is negative, $x^2 + 5x + 10 = 0$ has [ ] imaginary solutions.

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3 Evaluating the Discriminant A player throws a ball up and toward a wall that is 17 ft high. The height h in feet of the ball t seconds after it leaves the player's hand is modeled by $h = -16t^2 + 25t + 6$. If the ball makes it to where the wall is, will it go over the wall or hit the wall?

$h = -16t^2 + -5t + 6$

$17 = -16t^2 + 25t + 6$ **Substitute 17 for h.**

$0 = -16t^2 + 25t - \square$ **Write the equation in standard form.**

$a = \square, b = \square, c = \square$ **Find the values of a, b, and c.**

$b^2 - 4ac = (\square)^2 - 4(\square)(\square)$ **Evaluate the discriminant.**

$= \square - \square$ **Simplify.**

$= \square$

Since the discriminant is $\square$, the equation has no real $\square$. The ball will hit the wall.

Quick Check

1. Use the Quadratic Formula to solve each equation. Check your solutions.

a. $3x^2 - x = 4$

b. $-2x^2 = 4x + 3$

c. $4x^2 = 8x - 3$

d. $x^2 + 4x = 41$

2. Determine the type and number of solutions of each equation.

a. $x^2 + 6x + 9 = 0$

b. $x^2 + 6x + 10 = 0$

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Lesson 5-7

Completing the Square

Lesson Objectives	NAEP 2005 Strand: Algebra
1 Solving equations by completing the square	Topic: Equations and Inequalities
2 Rewriting functions by completing the square	Local Standards: _____

Vocabulary

Completing the square is a process for _____

_____

Examples

1 Solving a Perfect Square Trinomial Equation Solve $x^2 - 12x + 36 = 9$.

$x^2 - 12x + 36 = 9$

$(\square)^2 = 9$ Factor the trinomial.

$\square = \square$ Find the square root of each side.

$x - 6 = \square$ or $x - 6 = -\square$ Solve for x.

$x = \square$ or $x = \square$

2 Finding Complex Solutions Solve $x^2 + 6x + 12 = 0$.

$x^2 + 6x + 12 = 0$

$\left(\frac{\square}{2}\right)^2 = \square$ Find $\left(\frac{b}{2}\right)^2$.

$x^2 + 6x = \square$ Rewrite so all terms containing x are on one side.

$x^2 + 6x + \square = \square + \square$ Complete the square by adding 9 to each side.

$(\square)^2 = \square$ Factor the perfect square trinomial.

$x + \square = \pm\sqrt{\square}$ Find the square root of each side.

$x = \square \pm \sqrt{\square}$ Solve for x.

$= \square \pm \square\sqrt{3}$ Simplify.

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❸ Solving When $a \neq 1$ Solve $2x^2 + 7x - 1 = 0$.

$2x^2 + 7x - 1 = 0$

$x^2 + \frac{7}{2}x - \frac{1}{2} = 0$ — Divide each side by 2.

$x^2 + \frac{7}{2}x = \frac{\square}{\square}$ — Rewrite so all terms containing x are on one side.

$\left(\frac{\frac{7}{2}}{2}\right)^2 = \frac{\square}{\square}$ — Find $\left(\frac{b}{2}\right)^2$.

$x^2 + \frac{7}{2}x + \frac{\square}{\square} = \frac{1}{2} + \frac{\square}{\square}$ — Complete the square by adding $\frac{49}{16}$ to each side.

$\left(x + \frac{\square}{4}\right)^2 = \frac{\square}{16}$ — Factor the perfect square trinomial.

$x + \frac{7}{4} = \pm\frac{\sqrt{57}}{\square}$ — Find the square root of each side.

$x = \square \pm \square$ — Solve for x.

❹ Using Vertex Form The monthly profit P from the sales of rugs woven by a family rug-making business depends on the price r that they charge for a rug. The profit is modeled by $P = -r^2 + 500r - 59{,}500$. Write the function in vertex form. What is the maximum monthly profit, in dollars, determined by this model?

$P = -r^2 + 500r - 59{,}500$

$= -\left(r^2 - \square\right) - 59{,}500$ — Factor -1 from the first two terms.

$= -\left[r^2 - 500r + \left(\square\right)^2\right] - 59{,}500 + \left(\square\right)^2$ — Add and subtract $(-250)^2$ on the right side.

$= -\left(r - \square\right)^2 - 59{,}500 + \square$ — Factor the perfect square trinominal.

$= -\left(r - \square\right)^2 + \square$ — Simplify in vertex form.

The vertex is $\left(\square, \square\right)$. A price of \$ $\square$ per rug gives a maximum monthly profit of \$ $\square$.

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Quick Check

1. Solve $x^2 - 14x + 49 = 81$.

2. **a.** Check the solution to Example 2.

b. Solve $x^2 + 6x = -34$. Check your solution.

c. Solve $x^2 + 4x - 4 = 0$. Check your solution.

3. Solve each quadratic equation by completing the square.

a. $2x^2 + x = 6$

b. $2x^2 = 3x - 4$

4. **a.** Use the vertex form to find the vertex of $P = -\frac{1}{2}r^2 + 280r - 1{,}200$. Refer to Example 4 if necessary.

b. How do you know the vertex represents a maximum point?